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Numerical Solution of the Forward Magnetic Field Problem for Models with Irregular Polyhedron Discretization Taking into Account the "Demagnetization Effect"

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Abstract. A performance-effective numerical method for magnetic field calculation is proposed. The method works with irregular polyhedron discretization which enables us to construct models with magnetic objects of arbitrary shape. As a case study, a model of a well in plane-parallel layer is considered. The model is approximated with dense irregular grid, elements of which are polyhedrons. With the help of conjugate gradient method, we solve "demagnetization effect" equation and calculate total magnetic field on the plane above the well. For a well of 0.25m radius and 8m height "demagnetization effect" is of order of 2% relative to the field induced by the object placed in equivalent of Earth magnetic field.

INTRODUCTION

In numerical modeling of the magnetic field of objects which shape is poorly approximated by parallelepipeds (for example, a cylinder, as a prototype of a well), one can construct discretization of the magnetic medium model into irregular polyhedrons. In this paper, we propose a method for calculating the magnetic field from irregular polyhedrons, based on the transition to triangulation of polyhedron surface and the subsequent calculation of the contribution of individual triangular plate to the total field using closed-form expression (no approximation required). The algorithm was implemented in software using the Nvidia CUDA parallelization technology, which made it possible to carry out numerical experiments for models with a large number of discretization elements.

PROBLEM STATEMENT

Let $D = \{(x, y, z): x \in (-\infty, +\infty), y \in (-\infty, +\infty), z \in (z_1, z_2)\}$ be region of the model under consideration (plane-parallel layer) in Cartesian coordinate system $Oxyz$. The Oz axis is in down direction. The well is represented as a straight circular cylinder with base radius r and axis of symmetry collinear to Oz. The equation of the upper base plane is $z = z_1$, and of the lower is $z = \ h \le z_2$. Let us denote the inside of the cylinder as D_{int} and the outside as $D_{ext} = D \backslash D_{int}$. In the region D magnetic susceptibility is determined by K(a), a $\in D$. Let us denote the magnetization at the point a $\in D$ as $\mathbf{I}$(a). Then, total magnetic field strength $\mathbf{H}$ consists of two parts [1]:

$$\mathbf{H}(a) = \mathbf{H}^{prm}(a) + \mathbf{H}^{snd}(a)$$

$$\mathbf{H}^{snd}(a) = \frac{1}{4\pi} grad_a \int_D \mathbf{I}(q) grad_a \frac{1}{\|\boldsymbol{q} - \boldsymbol{a}\|} dV_q, \tag{1}$$

$$\mathbf{I} = \mathrm{K}\mathbf{H} + \mathbf{I}_n$$

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Where $\mathbf{H}^{prm}$ is external (primary) field, $\mathbf{H}^{snd}$ is body-generated field, $q \in D$ is point of integration, $\mathbf{I}_n$ is remanent magnetization.

Let's denote $\mathbf{I}^{prm} = \mathrm{K}\mathbf{H}^{prm} + \mathbf{I}_n$. If at all points $\mathrm{a} \in D$ we consider $\mathbf{H}^{prm}$, $\mathbf{I}_n$ and K known, then total magnetization **I** and field **H** can be found from the Fredholm equation of the second kind:

$$\mathbf{I}(a) = \mathbf{I}^{prm}(a) + \frac{\mathrm{K}(a)}{4\pi} grad_a \int_D \mathbf{I}(q) grad_a \frac{1}{\|\boldsymbol{q} - \boldsymbol{a}\|} dV.$$

Non-zero difference $\mathbf{I} - \mathbf{I}^{prm}$ called "demagnetization effect". The method proposed allows effective computation of the right side of the equation. It is used as basis for an iterative process for solving the equation. In our example presented below we will use simple iteration method. In cases where the effect of the demagnetization can be neglected and we can assume $\mathbf{I} = \mathbf{I}^{prm}$, the proposed method allows quickly calculate (1).

METHOD

We introduce a discretization of the region D with elements D_i: $D = \bigcup_i D_i$. Let $\mathbf{H}_i$ denote the contribution to the field $\mathbf{H}^{snd}$ of the element D_i with constant magnetization $\mathbf{I}_i$ up to a factor of $\frac{1}{4\pi}$. Then,

$$\mathbf{H}^{snd} = \frac{1}{4\pi}\sum_i \mathbf{H}_i\,.$$

We require the surface of D_i to have triangulation (we employed the same method for effective solving direct gravity problem [2]), we denote the set of triangles forming the surfaceD_i, as $\mathbb{S}(D_i)$, then

$$\mathbf{H}_i = -\sum_{S_k \in \mathbb{S}(D_i)} \left(\mathbf{I}_i, \mathbf{n}_{S_k}\right)\mathbf{U}_{S_k}\,,$$

where $\mathbf{n}_{S_k}$– external normal to S_k for the k'th triangle, $(\cdot,\cdot)$ – dot product, $\mathbf{U}_{S_k}$ – vector quantity numerically equal to the gravity field strength from the triangle S_k with density equal to $1/\gamma$, where γ is gravity constant. For an arbitrary triangle with radius vectors of vertices $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{A}_3$ we denote: $\mathbf{a}_i = \mathbf{A}_i/|\mathbf{A}_i|$, $\mathbf{A}_{ij} = \mathbf{A}_j - \mathbf{A}_i$, $\mathbf{a}_{ij} = \mathbf{A}_{ij}/\left|\mathbf{A}_{ij}\right|$, $\mathbf{n} = [\mathbf{A}_{31}, \mathbf{A}_{12}]/|[\mathbf{A}_{31}, \mathbf{A}_{12}]|$ – unit normal. Following

$$\mathbf{U} = 2\mathbf{n} \cdot \mathrm{arctg}\frac{(\mathbf{a}_1, [\mathbf{a}_2, \mathbf{a}_3])}{1 + (\mathbf{a}_3, \mathbf{a}_1) + (\mathbf{a}_1, \mathbf{a}_2) + (\mathbf{a}_2, \mathbf{a}_3)} + 2\left[\mathbf{n}, \mathbf{a}_{31} \cdot \mathrm{arth}\frac{|\mathbf{A}_{31}|}{|\mathbf{A}_3| + |\mathbf{A}_1|} + \mathbf{a}_{12} \cdot \mathrm{arth}\frac{|\mathbf{A}_{12}|}{|\mathbf{A}_1| + |\mathbf{A}_2|} + \mathbf{a}_{23} \cdot \mathrm{arth}\frac{|\mathbf{A}_{23}|}{|\mathbf{A}_2| + |\mathbf{A}_3|}\right],$$

where $[\cdot,\cdot]$ – vector product.

It should be noted that when applying this formula to S_k the numbering of the vertices of the triangle should be chosen so that from the side of the outer normal $\mathbf{n}_{S_k}$ the bypass of all vertices is counterclockwise.

MODEL DISCRETIZATION

Let us define region $F = \{(x, y, z)\colon x \in \left(x_{f1}, x_{f2}\right), y \in \left(y_{f1}, y_{f2}\right), z \in (z_{f1}, z_{f2})\}$, in which points we will calculate **H**. Instead of a region of infinite length D, we consider a finite region $D' = \{(x, y, z)\colon x \in (x_1, x_2), y \in (y_1, y_2), z \in (z_1, z_2)\}$. In order to eliminate boundary effects, we choose $x_1 \ll x_{f1}; x_{f2} \ll x_2; y_1 \ll y_{f1}; y_{f2} \ll y_2$.

We conduct discretization of $D^k = \left(x_{f1}, x_{f2}\right) \times \left(y_{f1}, y_{f2}\right) \times (z_1, z_2)$ to a set of polyhedrons. By choosing the diameter of the discretization, we can achieve the desired calculation accuracy $\mathbf{H}^{snd}$. Region $D' \backslash D^k$ we fill with a single polyhedron (or any minimally convenient amount). We consider its magnetization constant

Fig. 1 shows a discretization of a two-layer medium (side faces of the discretization elements are not shown). The central part of the image (with dense splitting) is the area over which the field is calculated. The remaining space is filled with 4 identical parallelepipeds (the upper and lower faces of 3 parallelepipeds are shown). The length of the parallelepipeds is 10^6 times of the length of the area with a dense partition.

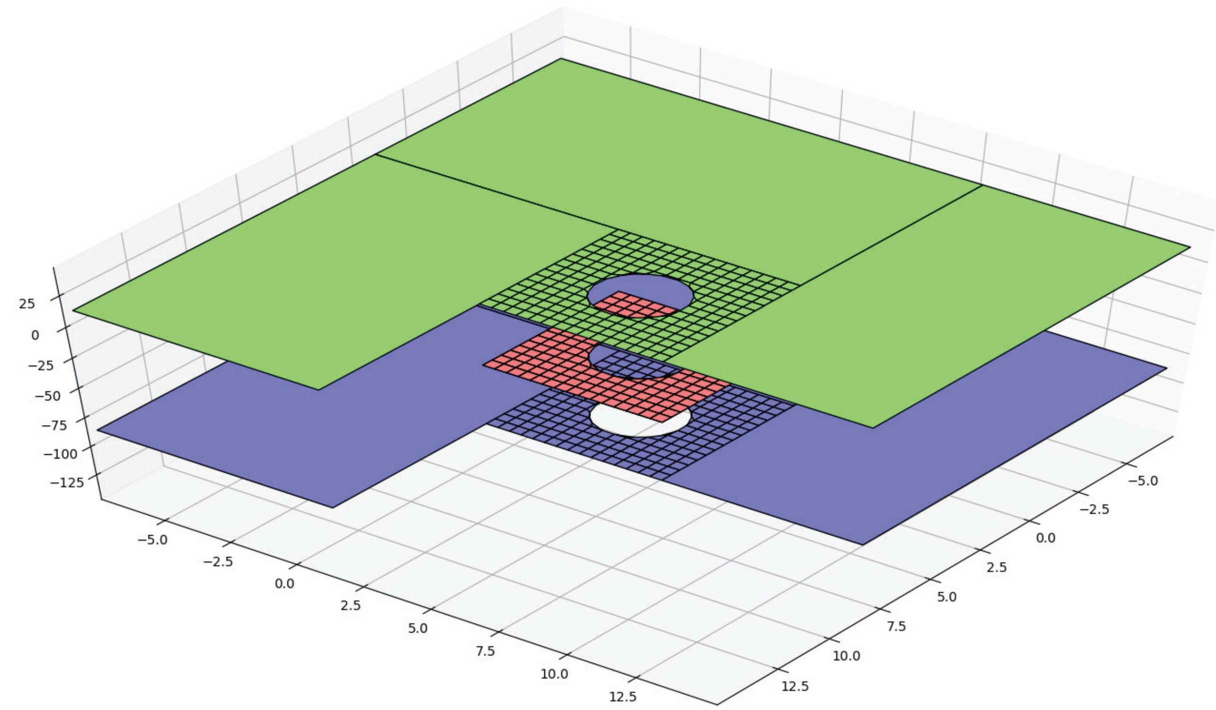

FIGURE 1. Visual representation of model discretization.

NUMERICAL EXPERIMENT

Consider parallelepiped in D of size 12 x 12 x 8 m., the axis of the cylinder of radius r = 0.25 m passes through the center of the upper and lower faces. In the parallelepiped K is set to 0.02. Inside of cylinder $K_C = 0$. In D magnetization is set as $\mathbf{I} = K\mathbf{H}^{prm}$, where $\mathbf{H}^{prm}$ = (14, 14, 35) A/m – constant external magnetic field. Model discretization: 480x480x20. The field is calculated on a plane with linear dimensions 8x8 m., located at a height of h = 0.25 m above the surface of the parallelepiped and centered relative to it. Filed discretization: 120x120 points. Fig. 2 shows the components of the calculated field $\mathbf{H}^{snd}$ (top row): X (min., max.: [-39; 53] mA/m) and Z (min., max.: [-105; 4] mA/m), as well as part of the field due to the "demagnetization effect" (bottom row), i.e. difference of $\mathbf{H}^{snd}$ calculated with and without the "demagnetization effect": X (min., max.: [-0.67; 0.76] mA/m) and Z (min., max.: [0; 1.67] mA/m). The average contribution of the "demagnetization effect" to the field $\mathbf{H}^{snd}$ is 1.8%. Error for "demagnetization effect" calculation was estimated with Runge rule and equals to 0.6%. Fig. 2 shows the modelling results.

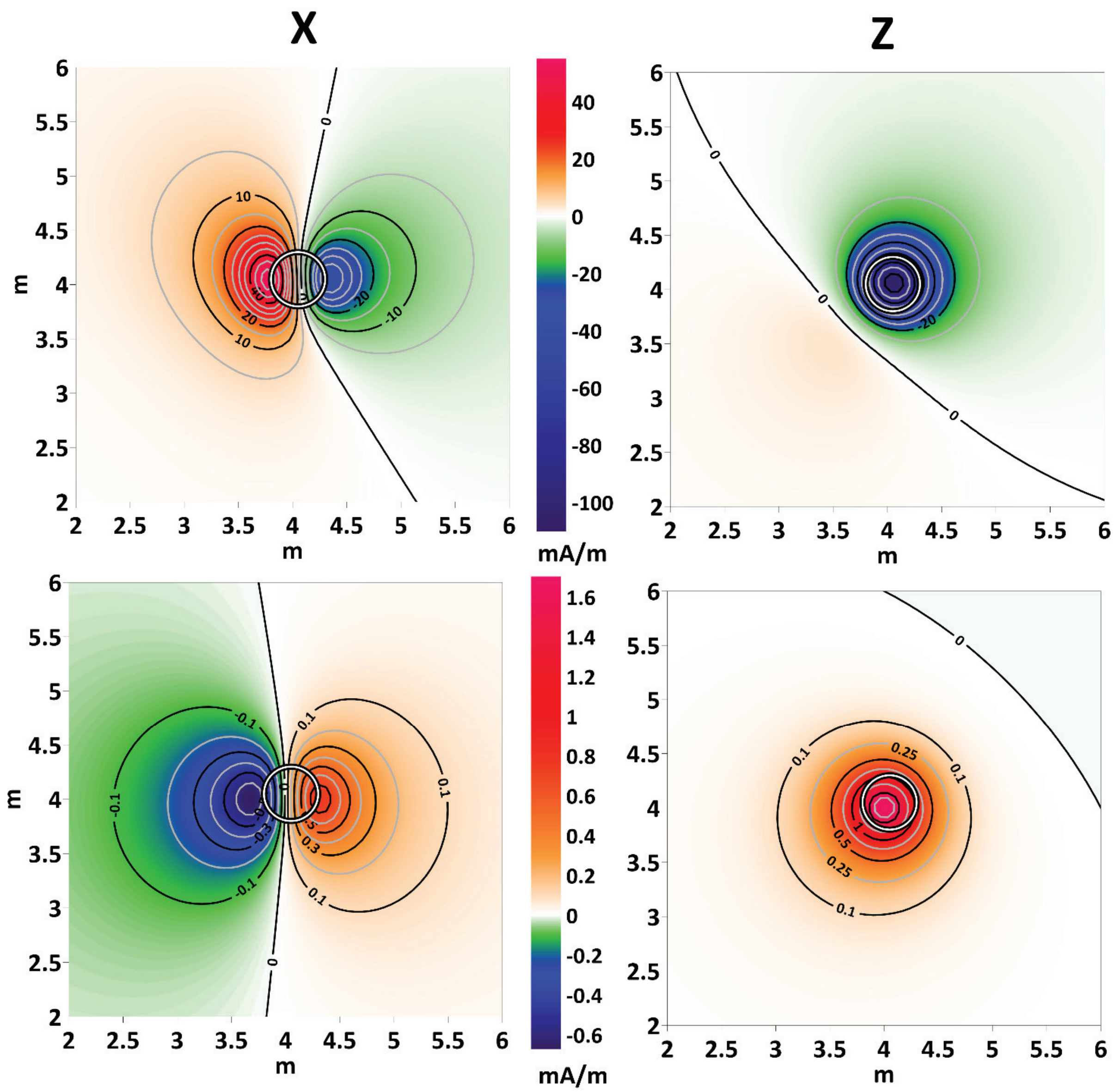

FIGURE 2. Calculated secondary (induced) magnetic field with "demagnetization effect" of the well model (upper row) and the part of the field caused by the demagnetization effect (lower row). The white circle indicates the projection of the well contour onto the field calculation plane.

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